

Veri biliminde
Karmaşıklığın fiziksel doğası

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Karmaşık Sistemler ve Veri Bilimi Çalıştayı
Bilgi Üniversitesi, İstanbul

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Veri (büyük veri) Sistemleri ve Karmaşık Sistemler ve kavramsallaştırma hataları

Veri (büyük veri) Sistemleri

- Çeşitlilik
- İç bağlantılı ağ yapısı (korelasyonlar) (!)
- Örüntü (Pattern) (!)
- Hacim
- Non-linearity (!)
- Self-organization (!)
- Sistem oluşturma (!)
- Sistem + düzensizlik
- Heterojen yapı
- Sınırlı/random/autonom

Kompleks Sistemler

- Çeşitlilik
- İç bağlantılı ağ yapısı (korelasyonlar)
- Örüntü (Pattern)
- Hacim (!)
- Non-linearity
- Self-organization
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- Sınırlı/random/autonom

Karmaşıklık Nasıl ölçülür?

ve yetersiz analizler

- **Algorithmic complexity (Kolmogorov-Chaitin):** Kolmogorov, A.N.: Three approaches to the definition of quantity of information. *Probl. Inform. Theory* 1, 3-11 (1965); Chaitin, G.J.: On the length of programs for computing finite binary sequences. *J. Assoc. Comput. Mach.* 13, 547-569 (1966).
- **Statistical complexity (Shannon, Renyi Entropies and LMC Complexity) :** Shannon, C.E., Weaver, W.: *The Mathematical Theory of Communication*. University of Illinois Press, Urbana, Illinois (1949); Renyi, A.: *Probability Theory*. North-Holland, Amsterdam (1970); Calbet, X., Lopez-Ruiz, R.: Extremum complexity distribution of a monodimensional ideal gas out of equilibrium. *Physica A* 382, 523-530 (2007).

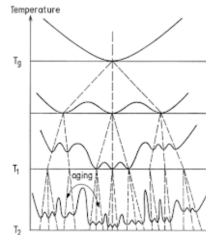
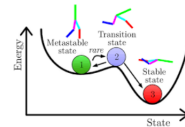
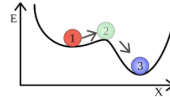
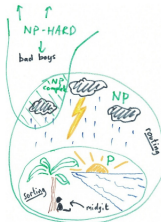
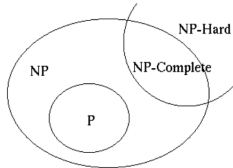
Algoritmik Karmaşıklık ve kavramsallaştırma hataları

P and NP

P: Problems solvable in polynomial time.

NP: Problems verifiable in polynomial time

NPC: Hardest problems in NP.



Düzen - Düzensizlik - Karmaşa (!)

İstatistiksel Karmaşıklık

Complexity in a network (Sporns, 2011)

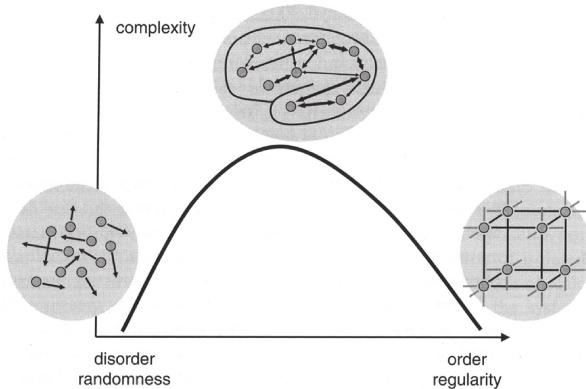


Figure 13.1

Complexity as a mixture of order and disorder. This schematic diagram illustrates different levels of complexity present in highly disordered systems (“gas”), highly ordered systems (“crystal”), and systems that combine elements of order and disorder (“brain”).

İstatiksel Karmaşıklık Tanımları

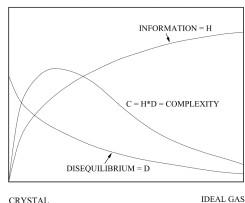


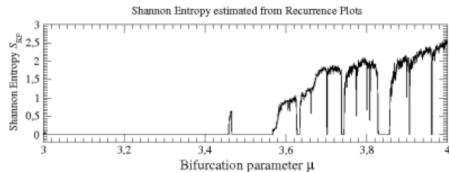
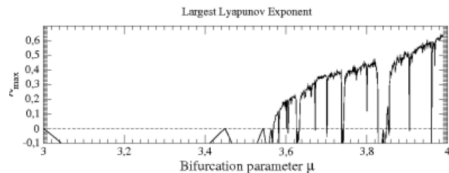
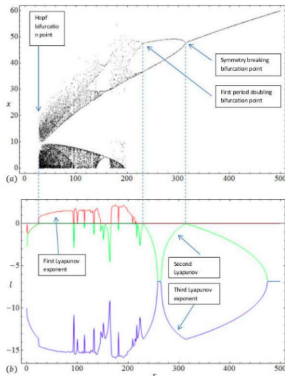
Fig. 1.1 Sketch of the intuitive notion of the magnitudes of "information" (H) and "disequilibrium" (D) for the physical systems and the behavior intuitively required for the magnitude "complexity". The quantity $C = H \cdot D$ is proposed to measure such a magnitude.

Figure: R. Lopez-Ruiz, H. Mancini, X. Calbet: A statistical measure of complexity, arXiv:1009.1498v1

$$C = H \cdot D = - \left(K \sum_{i=1}^N p_i \log p_i \right) \cdot \left(\sum_{i=1}^N \left(p_i - \frac{1}{N} \right)^2 \right) \quad (1)$$

$$C = H \cdot D = - \left(K \int_{-\infty}^{\infty} p(x) \log p(x) \right) \cdot \left(\int_{-\infty}^{\infty} p^2(x) dx \right) \quad (2)$$

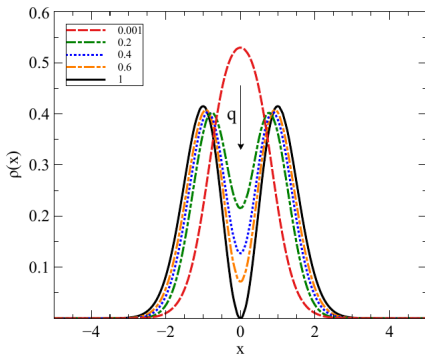
İstatiksel Karmaşıklık Tanımı Ne İşe Yarar? Örnek-1



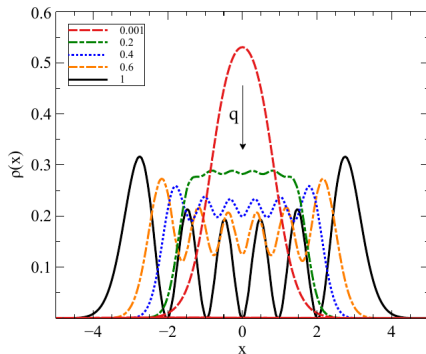
C. Letellier, Estimating the Shannon entropy: Recurrence plots versus symbolic dynamics, Physical Review Letters, 96, 254102, 2006.

İstatiksel Karmaşıklık Tanımı Ne İşe Yarar? Örnek-2

$$|n\rangle = \Psi_n(x) = \frac{\exp(-\frac{x^2}{2} + \frac{3}{2}i\alpha x)}{\pi^{\frac{1}{4}} i^n (1 - \exp(-2\alpha^2))^{\frac{n}{2}} \sqrt{[n]_q!}} \sum_{k=0}^n \frac{(-1)^k [n]_q!}{[k]_q! [n-k]_q!} \exp(2i\alpha x(n-k) - k\alpha^2) \quad (3)$$



(a) $n = 1$



(b) $n = 5$

Figure 1: q dependence of the probability distribution under q -deformed harmonic oscillator for levels $n = 1$ and $n = 5$.

İstatiksel Karmaşıklık Tanımı Ne İşe Yarar? Örnek-2 (devam...)

$$|n\rangle = \Psi_n(x) = \frac{\exp(-\frac{x^2}{2} + \frac{3}{2}i\alpha x)}{\pi^{\frac{1}{4}} i^n (1 - \exp(-2\alpha^2))^{\frac{n}{2}} \sqrt{[n]_q!}} \sum_{k=0}^n \frac{(-1)^k [n]_q!}{[k]_q! [n-k]_q!} \exp(2i\alpha x(n-k) - k\alpha^2) \quad (4)$$

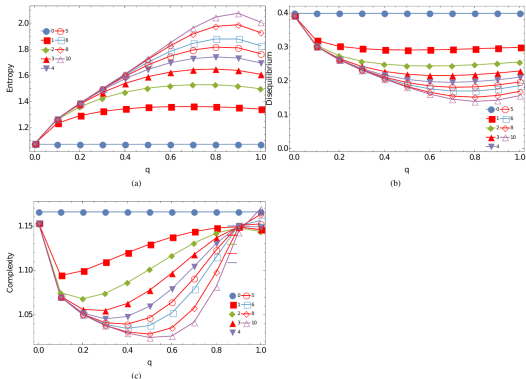


Figure 2: q dependence of the Shannon information entropy (a), disequilibrium (b) and complexity (c) for -deformed harmonic oscillator for energy levels in $[0,10]$ where $n=7,9$ have omitted for preventing messiness.

İstatiksel Karmaşıklık Tanımı Ne İşe Yarar? Örnek-3

$$\Psi_n(x) = N_n \exp \left\{ -\alpha \left(\lambda q - n - \frac{1}{2} \right) x - \lambda e^{-\alpha x} \right\} L_n^{2(\lambda q - n - \frac{1}{2})} (2\lambda e^{-\alpha x}) \quad (5)$$

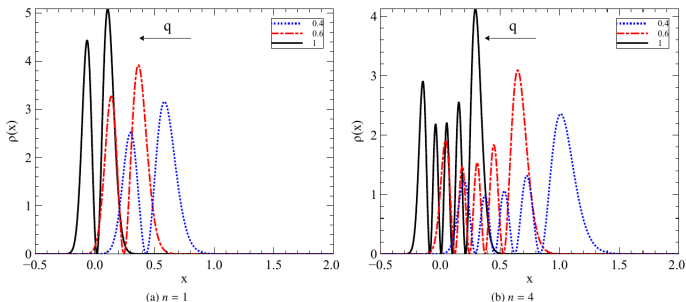
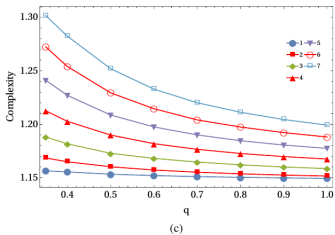
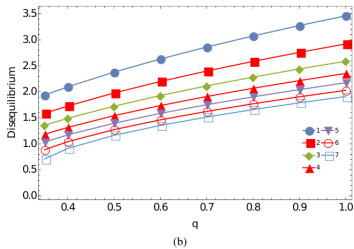
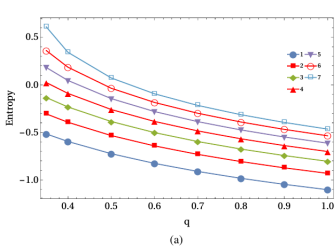


Figure 3: q dependence of the probability distribution under q -deformed Morse potential for levels $n = 1$ and $n = 5$ of HCl molecule.

İstatiksel Karmaşıklık Tanımı Ne İşe Yarar? Örnek-3 (devam...)

$$\Psi_n(x) = N_n \exp \left\{ -\alpha \left(\lambda q - n - \frac{1}{2} \right) x - \lambda e^{-\alpha x} \right\} L_n^{2(\lambda q - n - \frac{1}{2})} (2\lambda e^{-\alpha x}) \quad (6)$$



Önemli problemler

- Sistemlerde gömülü örüntüler belirlenebilir mi?
- Sistemin dinamiği tahmin edilebilir mi?
- ???
- Karmaşıklık teori ve yöntemleri bir işe yarar mı?
- Kritik geçişlerin belirlenmesi
- (...)

Beni dinlediğiniz için teşekkürler...