

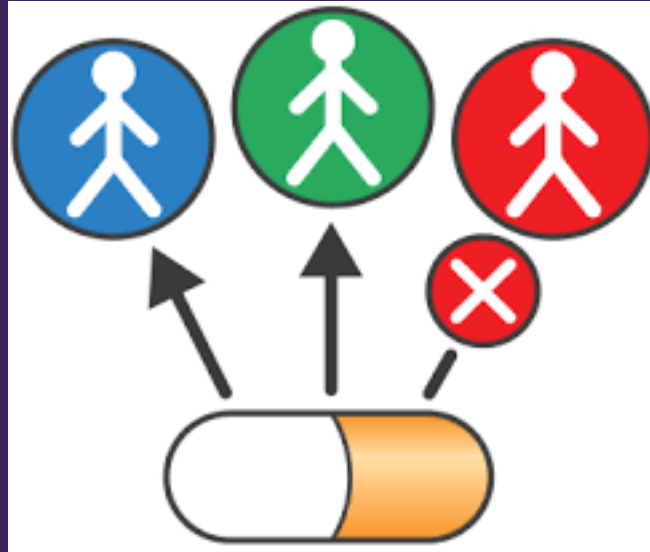
Complex Systems and its Applications

Spontaneous emergence of order

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Personalised Treatments

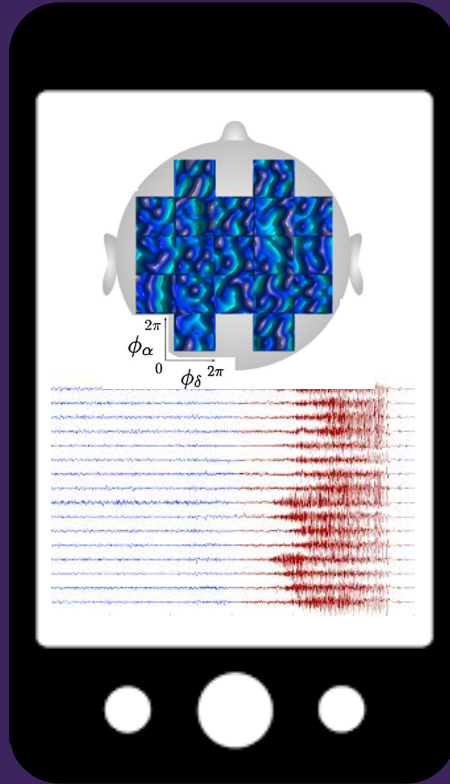
Personalised Treatment



Access to data



Access to data



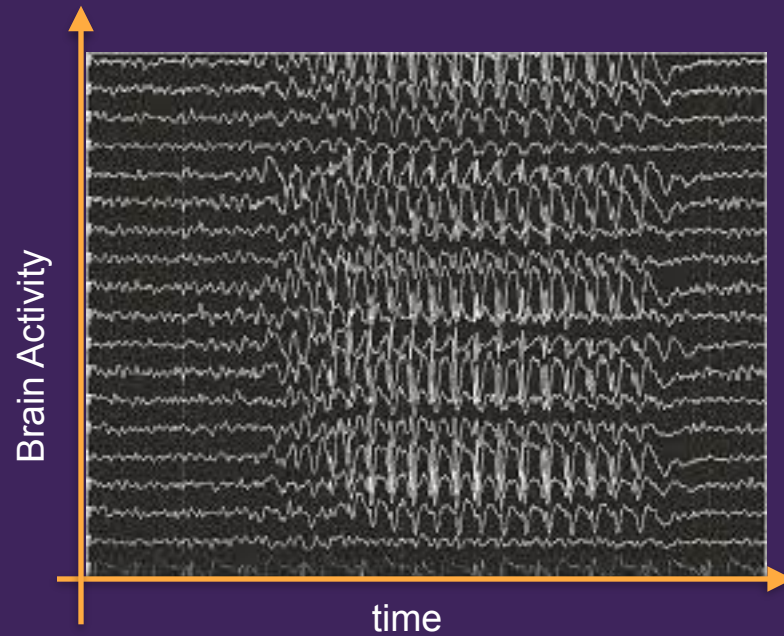
Understanding its behaviour



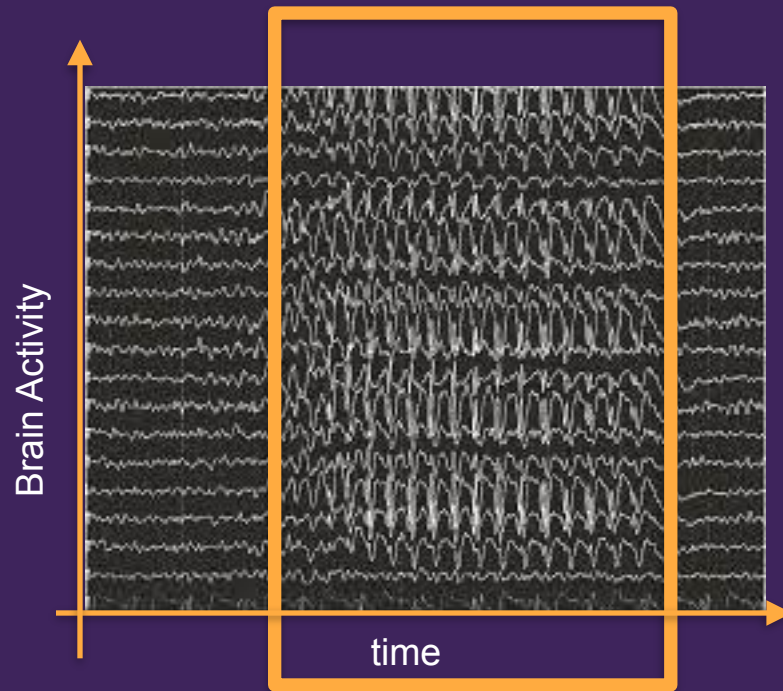
Too much data...



Uncovering patterns

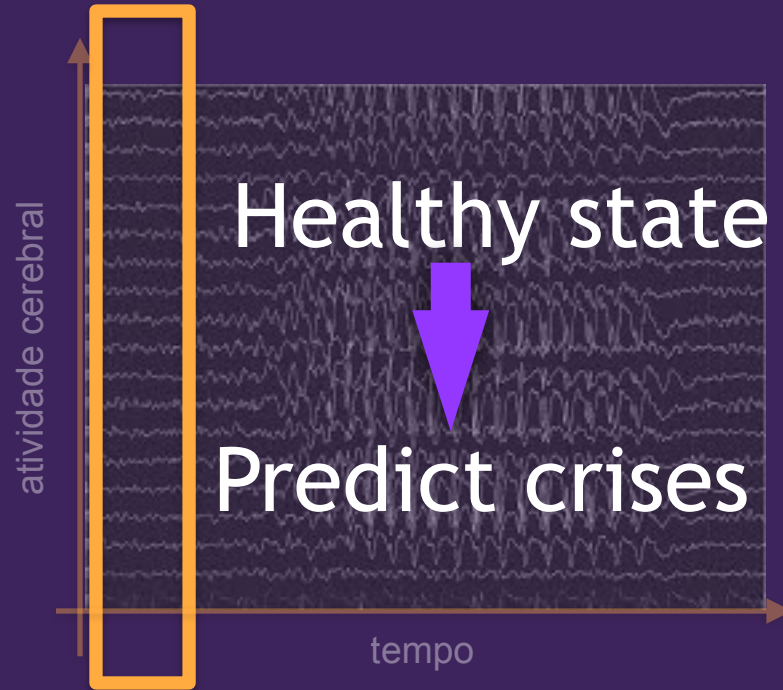


Uncovering patterns



Seizure

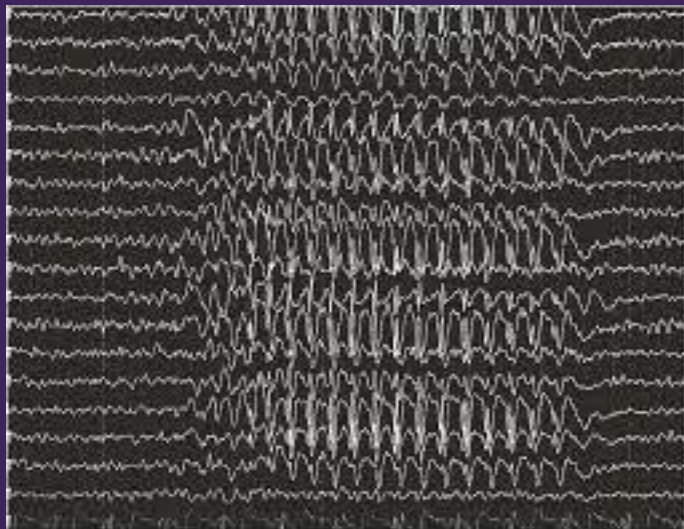
Uncovering patterns



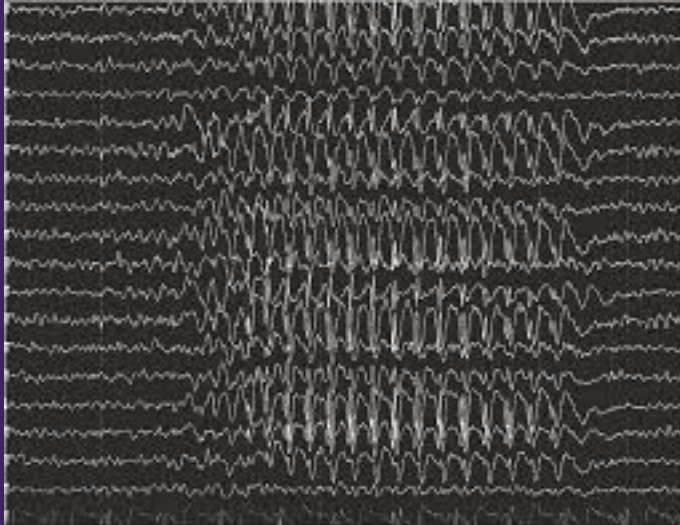
Personalised treatment

What is the best treatment for you?

Uncovering rules



Uncovering rules



$$\lim_{x \rightarrow 0} \frac{e^{2x}-1}{5x} = \frac{2}{5} \quad \lambda_2 = \sqrt{4} = 2 = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$$

$$z = \frac{1}{x} \arcsin \frac{\sqrt{z}}{z} \quad \frac{\partial z}{\partial x} = 2, \frac{\partial z}{\partial y} = 0 \quad \int R(x, \frac{P(x)+b}{C(x+d)}) dx$$

$$x_2 = \begin{pmatrix} -x \\ -y \\ -z \end{pmatrix} \quad C = \begin{pmatrix} 0, 1 \\ 1, 0 \end{pmatrix} \quad \lim_{n \rightarrow +\infty} \frac{\sqrt{n^2+1}+n}{\sqrt{3n^2+2n-1}} \quad F_0 = 2 \times yz - 1 = 1 \quad \eta_1 = \lambda_1^2 - 3\lambda_2 + 1 = 0$$

$$x \in \mathbb{R} \quad \begin{pmatrix} \alpha+\beta+1 \\ \alpha \\ \beta \end{pmatrix} \rightarrow \vec{n} = (F_x; F_y; F_z) \quad A = \begin{pmatrix} x, 1+x^2, 1 \\ y, 1+y^2, 1 \\ z, 1+z^2, 1 \end{pmatrix} \quad x=0, y=1, z=2$$

$$\iiint_M z dx dy dz = \int_0^{2\pi} \int_0^2 \int_0^1 r^2 dr d\varphi dz = \int_0^{2\pi} \int_0^2 \frac{1}{3} r^3 \Big|_0^1 d\varphi dz = \frac{1}{3} \int_0^{2\pi} \int_0^2 1 d\varphi dz = \frac{1}{3} \cdot 2\pi \cdot 2 = \frac{4\pi}{3}$$

$$\frac{2x}{x^2+2y^2} = 2 \frac{\sin x}{x} \leq \frac{x}{x} = 1 \quad \beta(A) = \sqrt{10} \pi$$

$$x^4 x^2 + y^2 + z^3 + x y z - C = 0 \quad x_1 = -1, x_2 = -2, x_3 = 7, p \in \mathbb{R}$$

$$2 \arctan x - x = 0, I = (1, 10)$$

$$A+B+C=8 \quad \cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1 \quad y' = \frac{1}{x+2} = 0, y(0) = 1$$

$$-3A-7B+2C = -10, 3 \quad k \neq 0, \mu \neq 0 \quad \lim_{x \rightarrow 0} \frac{e^{2x}-1}{5x} = \frac{2}{5} \quad \langle \lambda, \beta, \gamma \in \mathbb{C} \quad g \circ df = \left(\frac{\partial f}{\partial x}; \frac{\partial f}{\partial y} \right)$$

$$-18A+6B-3C = 15 \quad e^{-x} \cdot xy z = e, A[0, e; 1] \quad y(0) = 1 \quad A = [1, 0; 3]$$

$$\int_{-\pi/2}^{\pi/2} \sin^4 x \cdot \cos^3 x dx \quad y = \sqrt{x+1}, x = \tan t \quad \int 3x^2 + 16x^{-0.12} dx \quad \lim_{n \rightarrow +\infty} \left(1 + \frac{2}{n}\right)^n \quad f(x) = 2^{-x} + 1, \epsilon = 0.005$$

Uncovering rules

$$\begin{aligned}
 &\lim_{x \rightarrow 0} \frac{e^{2x} - 1}{5x} = \frac{2}{5}; \quad \lambda_2 = i\sqrt{14}; \quad B = \begin{pmatrix} 3 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} \\
 &z = \frac{1}{x} \arcsin \frac{\sqrt{z}}{z} \quad \frac{\partial z}{\partial x} = 2; \quad \frac{\partial z}{\partial y} = 0 \int R(x, \frac{y \pm mb}{C \pm md}) dx \quad \frac{2x}{x^2 + 2y^2} = 2 \frac{\sin x}{x} \leq \frac{x}{x} = 1 \quad \delta(A) = \sqrt{10}i \\
 &x^4 x^2 + y^3 + z^3 + xy^2 - C = 0 \quad x^4 x^2 + y^3 + z^3 + xy^2 - C = 0 \quad x_1 = -1, x_2 = -1, x_3 = 7, p \in \mathbb{R} \\
 &x_2 \begin{pmatrix} -x \\ \beta \\ -\beta \end{pmatrix} C = \begin{pmatrix} 0, 1 \\ 1, 0 \end{pmatrix} \quad \lim_{n \rightarrow \infty} \frac{\sqrt{n^2 + 1} + n}{\sqrt{3n^2 + 2n - 1}} \quad F_3 = 2 \times yz - 1 = 1 \quad \eta_1 = \lambda^2 - 3\lambda + 1 = 0 \\
 &x \in \mathbb{R} \quad \begin{pmatrix} \alpha + \beta + \gamma \\ \alpha \\ \beta \end{pmatrix} \vec{n} = (F_x; F_y; F_z) \quad A = \begin{pmatrix} x & 1 + x^2 & 1 \\ y & 1 + y^2 & 1 \\ z & 1 + z^2 & 1 \end{pmatrix} \quad x = 0, y = 1, z = 2 \\
 &\iiint_{\mathcal{A}} z \, dy \, dz \, dx = \int_0^1 \int_0^2 \int_0^1 (1 + y^2 + z^2) \, dx \, dy \, dz \in \mathcal{B} \quad (1, 0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}) \\
 &2 \arctan x - x = 0, \quad I = (1, 10) \\
 &\frac{\partial f}{\partial x} = 16 - x^2 + 16y^2 - 4z = 0 \quad \cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1 \quad y' = \frac{1 - \sqrt{1 - y^2}}{x + 2} = 0; \quad y(0) = 1 \\
 &A + B + C = 8 \quad \lim_{x \rightarrow 0} \frac{e^{2x} - 1}{5x} = \frac{2}{5} \quad \alpha, \beta, \gamma \in \mathbb{C} \quad g \circ df = \begin{pmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{pmatrix} \\
 &-3A - 7B + 2C = 10, 3 \quad (1 \cdot e^x) \gamma \gamma' = e^x \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 0 \\
 &-18A + 6B - 3C = 15 \quad e^z - xy^2 = e; \quad A[0, e, 1] \quad y(y) = 1 \quad A = [1, 0, 3] \\
 &\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^4 x \cdot \cos^3 x \, dx \quad \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 3x^2 + 16x^{-0.12} \, dx \quad \lim_{n \rightarrow \infty} (1 + \frac{2}{n})^n \quad f(x) = 2^{-x} + 1, \epsilon = 0.005
 \end{aligned}$$



Predict transitions

Treatments

Big challenge: Many parts



A 400 years old journey

from parts to the whole

In the Asian banks

Philip Laurent paper in Science in 1917

A phenomenon discussed for 300 years

single



lots



In the Asian banks



In the Asian banks

tendency to order

only an illusion?

In the Asian banks

1917 to 1930 we had 20 papers

In the Asian banks

Does Nature longs for order?

Nature longs for order

John Buck in 1960

Nature longs for order

Buck realised the order was emergent

first pairs and trios

In the lab

Buck confirmed the phenomenon

Christiaan Huygens

First to use mathematics

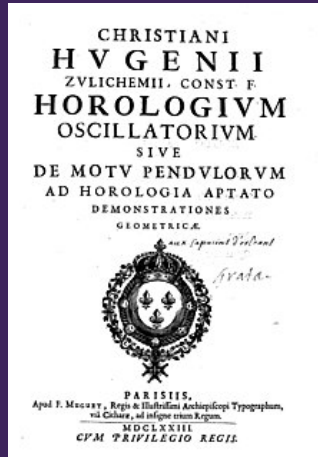


Christiaan Huygens



Pendulum clock

Christiaan Huygens



Christiaan Huygens

But



Christiaan Huygens

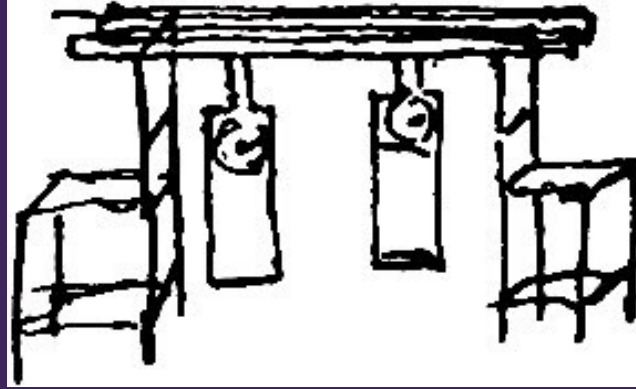


Christiaan Huygens

in the mist of dreams

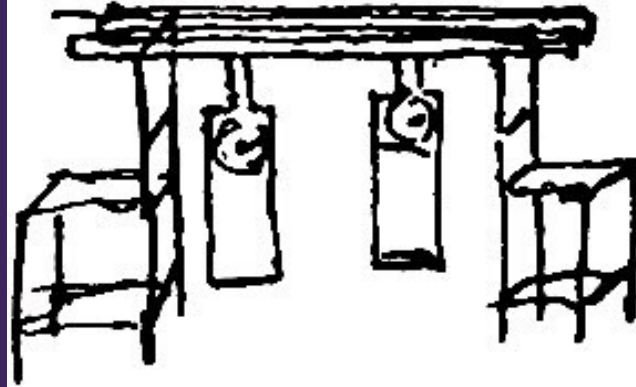


Christiaan Huygens



Pendulum in sync

Christiaan Huygens



Pendulum in sync

Christiaan Huygens

Modern version

Christiaan Huygens



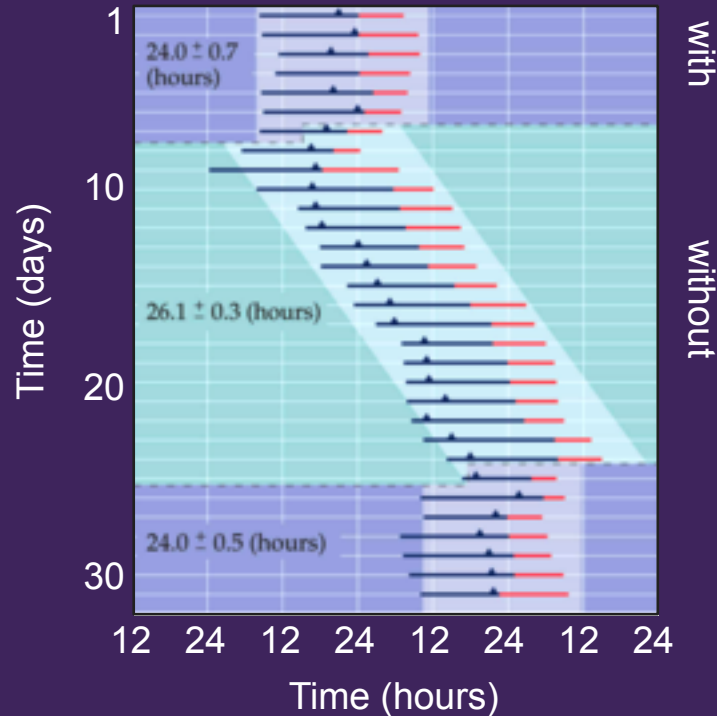
Tendency towards order

menstrual cycle



Tendency towards order

Circadian



Tendency towards order



Mystery

Why does order emerges spontaneously?

Tendency towards order

Mathematics describes
all these phenomena

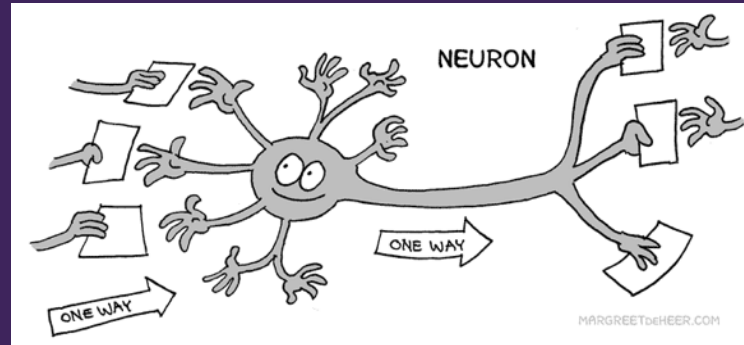
Tendency towards order

Predict when order
will appear

Order is not always good

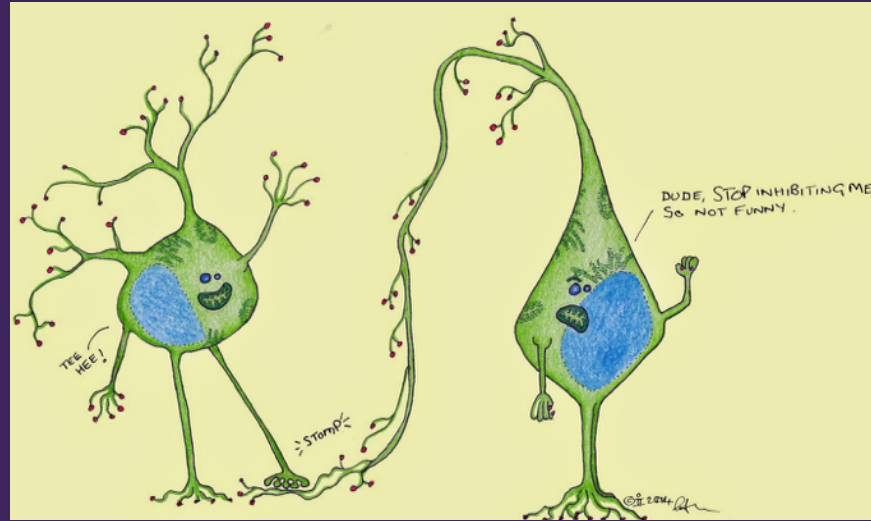
In the brain

neurons

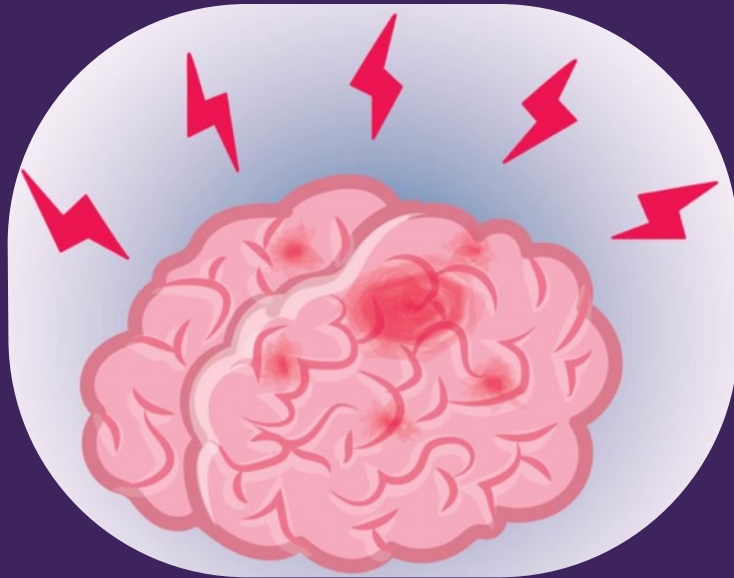


In the brain

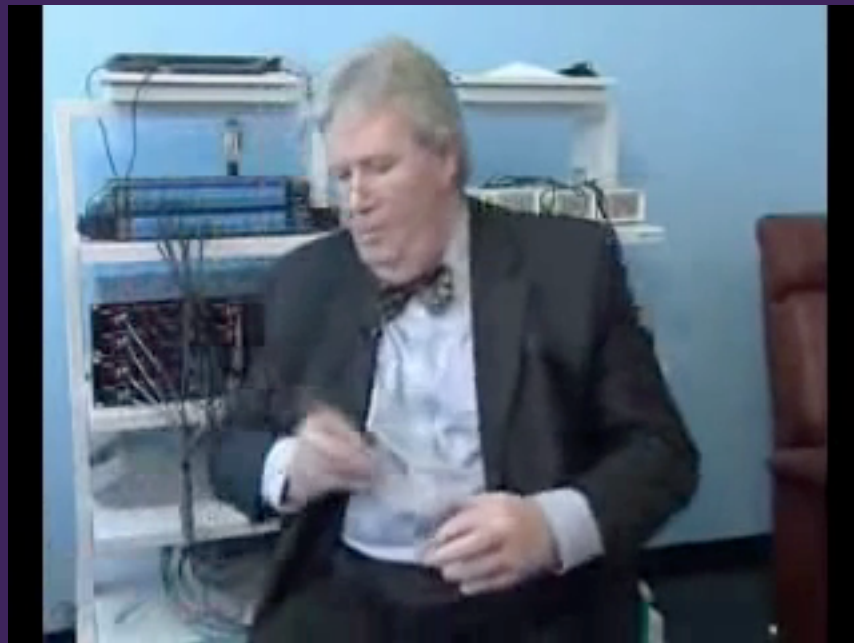
neurons talk to each other



neurons can synchronize



Parkinson

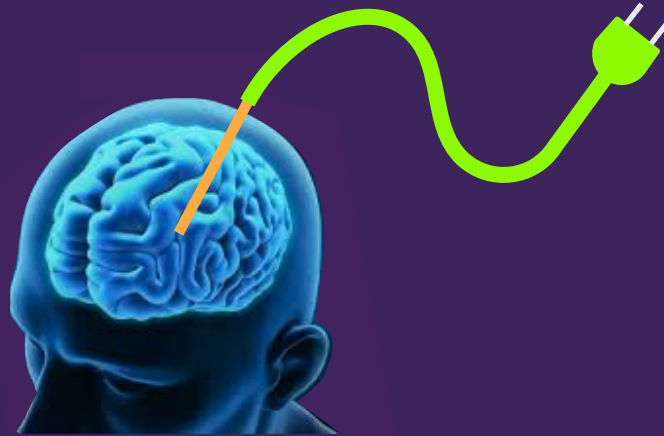


Parkinson



electrical activity in the brain

Parkinson



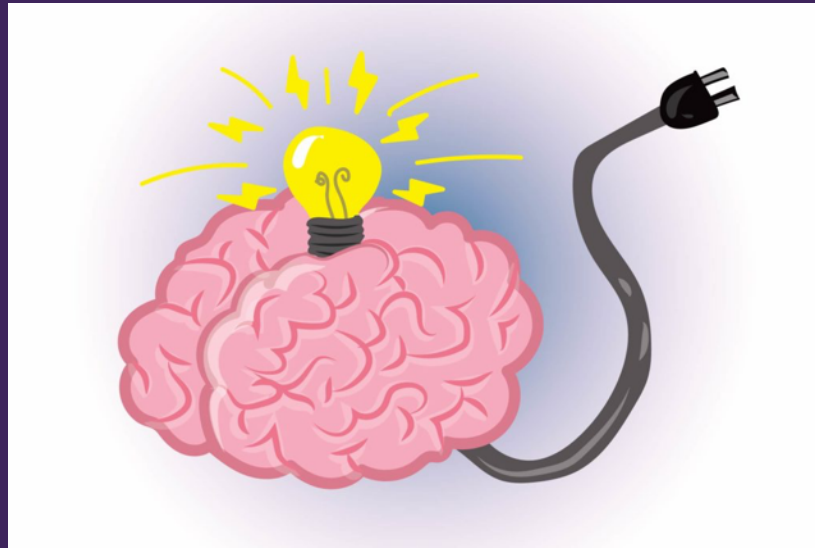
Why not to provide a small shock

Parkinson



Parkinson

Shocking all day long



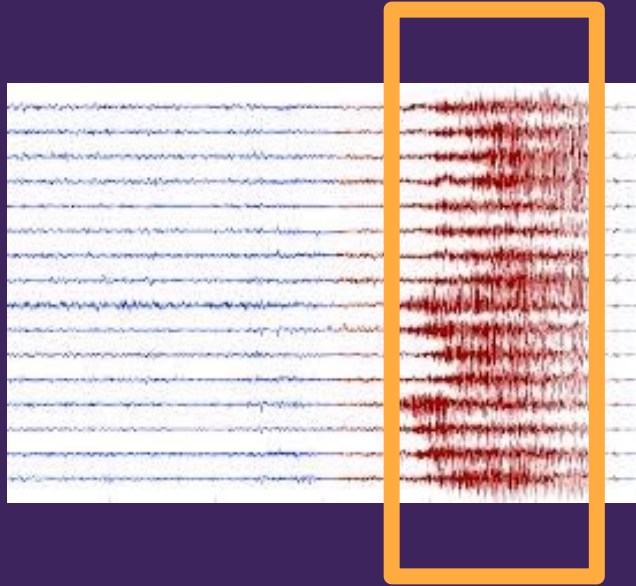
Parkinson

Obtain the equations that
rule the system

Parkinson

Least intervention

Epilepsy



Synchronization

A study case



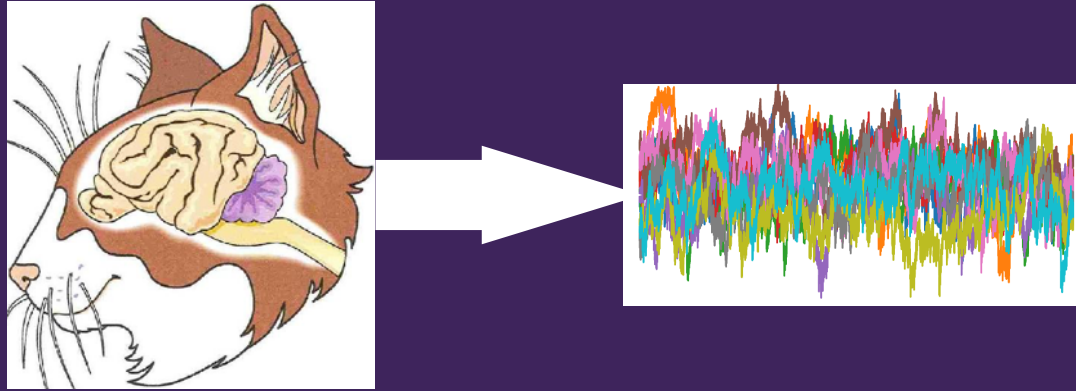
Together with Deniz



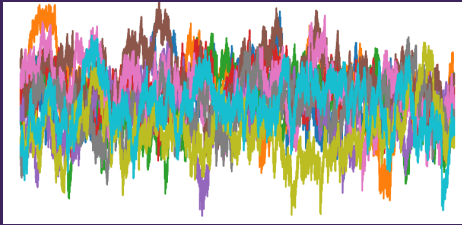
to



Simulate the system in a computer



From data to rules



$\lim_{x \rightarrow 0} \frac{e^{2x}-1}{5x} = \frac{2}{5}$, $\lambda_2 = i\sqrt{14}$, $B = \begin{pmatrix} 3 & 1 & -1 \\ 2 \end{pmatrix}$

$z = \frac{1}{x} \arcsin \frac{\sqrt{2}}{2}$, $\frac{\partial z}{\partial x} = -2$, $\frac{\partial z}{\partial y} = 0$, $\int \mathbb{R}(x, \sqrt{\frac{ax+b}{cx+d}}) dx$

$x_2 \begin{pmatrix} x \\ y \\ z \end{pmatrix} C = \begin{pmatrix} 0, 1 \\ 1, 0 \end{pmatrix}$, $\lim_{n \rightarrow \infty} \frac{\sqrt{n^2+1}+n}{3\sqrt{3n^2+2n-1}}$, $F_2 = 2xy^2 - 1 = 1$, $\eta_1 = \lambda^2 - 3\lambda_1 + 1 = 0$

$X \in \mathbb{R}$, $X_i = \begin{pmatrix} \alpha + \beta + \gamma \\ \beta \end{pmatrix}$, $\vec{n} = (F_x, F_y, F_z)$, $A = \begin{pmatrix} x, 1+x^2, 1 \\ y, 1+y^2, 1 \\ z, 1+z^2, 1 \end{pmatrix}$, $x=0, y=1, z=2$, $V_{i+1} = V_i + b_i k_i \sum_{j=1}^n (h^{(j)} - V_j)^2$

$\iiint_M z dx dy dz = \int_0^{2\pi} \left(\int_0^2 \left(\int_0^1 r r dr \right) d\theta \right) d\varphi$, $2 \arctan x - x = 0$, $I = (1, 10)$

$\lambda x - y + z = 1$, $x + y + z = \lambda$, $X_1 = \begin{pmatrix} 2p \\ -p \end{pmatrix}$, $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$, $y' = \frac{\sqrt{y}}{x+2} = 0$, $y(0) = 1$

$A+B+C=8$, $-3A-7B+2C=-10,3$, $-18A+6B-3C=15$, $k|+b| \neq 0$, $p \neq 0$, $\lim_{x \rightarrow 0} \frac{e^{2x}-1}{5x} = \frac{2}{5}$, $\alpha, \beta, \gamma \in \mathbb{C}$, $g \circ df = \begin{pmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \end{pmatrix}$

$e^2 - xy^2 = e$, $A[0, e, 1]$, $V(0) = 1$, $A = [1, 0, 3]$

$\int_{-\pi/2}^{\pi/2} \sin^4 x \cdot \cos^3 x dx$, $y = \sqrt[3]{x+1}$, $x = \lg t$, $\int 3x^2 + 166x - 0,17 dx$, $\lim_{n \rightarrow \infty} \left(1 + \frac{3}{n}\right)^n$, $f(x) = 2^{-x} + 1$, $\epsilon = 0.005$

From data to rules

Handwritten mathematical notes on a blackboard background, containing various calculus and algebra problems and solutions:

- $\lim_{x \rightarrow 0} \frac{e^{2x}-1}{5x} + \frac{2}{3}$; $\lambda_2 = i\sqrt{14}$; $B = \begin{pmatrix} 2 & 1 & 0 \\ 3 & 1 & 2 \end{pmatrix}$
- $z = \frac{1}{x} \arcsin \frac{\sqrt{z}}{z}$; $\frac{\partial z}{\partial x} = 2$; $\frac{\partial z}{\partial y} = 0$; $\int_{\mathbb{R}} (x, \sqrt{\frac{2x+1}{x+1}}) dx$
- $\frac{2x}{x^2+2y^2} = 2 \frac{\sin x}{x} \leq \frac{x}{x} = 1$; $\delta(\rho) = \sqrt{10}$
- $x^4 x^2 + y^3 + z^3 + x y z - C = 0$; $x_1 = -1, x_2 = -1, x_3 = 7, p \in \mathbb{R}$
- $F_3 = 2x y z - 1 = 1$; $\eta_1 = \lambda^2 - 3\lambda + 1 + 0$
- $x_2 = \begin{pmatrix} x \\ \beta \\ -\beta \end{pmatrix}$; $C = \begin{pmatrix} 0, 1 \\ 1, 0 \end{pmatrix}$; $\lim_{n \rightarrow \infty} \frac{\sqrt{n^2+1} + n}{\sqrt{3n^2+2n-1}}$; $A = \begin{pmatrix} x & 1+x^2 & 1 \\ y & 1+y^2 & 1 \\ z & 1+z^2 & 1 \end{pmatrix}$; $x=0, y=1, z=2$
- $X \in \mathbb{R}$; $X_1 = \begin{pmatrix} \alpha + \beta + \gamma \\ \beta \end{pmatrix}$; $\vec{n} = (F_x, F_y, F_z)$; $Y_{i+1} = Y_i + k_i \Delta t (F(X_i, Y_i))$
- $\iiint_M z dx dy dz = \int_0^{2\pi} \int_0^2 \int_0^1 r^2 dr d\phi d\psi \cos \phi = (1, 0, \frac{2}{\sqrt{2}}, \frac{1}{\sqrt{2}})$
- $2 \arctan x - x = 0$; $I = (1, 10)$
- $x x - y + z = 1$; $x + y + z = \lambda$; $X_1 = \begin{pmatrix} 2p \\ -p \\ 0 \end{pmatrix}$; $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$; $Y' = \frac{\sqrt{y}}{x+2} = 0$; $Y(0) = 1$
- $A+B+C=8$; $3A-7B+2C=10, 3$; $k \neq 0$; $\lim_{x \rightarrow 0} \frac{e^{2x}-1}{5x} + \frac{2}{3}$; $\langle \beta, \gamma \rangle \in C$; $\text{grad} f = (\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y})$
- $-18A+6B-3C=15$; $e^2 - x y z = e$; $A \in [0, e; 1]$; $Y(0) = 1$; $A = [1, 0, 3]$
- $\int_{-\pi/2}^{\pi/2} \sin^4 x \cdot \cos^3 x dx$; $y = \sqrt{x+1}$; $x = \log t$; $\int \frac{1}{3x^2+166x-913} dx$; $\lim_{h \rightarrow \infty} (1 + \frac{2}{h})^h$; $f(x) = 2^{-x} + 1, C = 0.005$



From rules to prediction

From data to rules

Handwritten mathematical notes on a blackboard background, showing various calculus and algebra problems and solutions. The notes include:

- $\lim_{x \rightarrow 0} \frac{e^{2x}-1}{5x} + \frac{2}{3}$, $\lambda_2 = i\sqrt{14}$, $B = \begin{pmatrix} 2 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$
- $z = \frac{1}{x} \arcsin \frac{\sqrt{z}}{2}$, $\frac{\partial z}{\partial x} = 2$, $\frac{\partial z}{\partial y} = 0$, $\int_{\mathbb{R}} f(x, \sqrt{\frac{2x+1}{x+1}}) dx$
- $\frac{2x}{x^2+2y^2} = 2 \frac{\sin x}{x} \leq \frac{x}{x} = 1$, $\delta(\rho) = \sqrt{10}$
- $x^4 x^2 + y^3 + z^3 + x y z - C = 0$, $x_1 = -1, x_2 = -1, x_3 = 7, p \in \mathbb{R}$
- $F_3 = 2xy - 1 = 1$, $\eta_1 = \lambda^2 - 3\lambda + 1 + 0$
- $x_2 = \begin{pmatrix} x \\ y \\ z \end{pmatrix} C = \begin{pmatrix} 0, 1 \\ 1, 0 \end{pmatrix}$, $\lim_{n \rightarrow \infty} \frac{\sqrt{n^2+1} + n}{\sqrt{3n^2+2n-1}}$
- $x \in \mathbb{R}$, $X_1 = \begin{pmatrix} \alpha + \beta + \gamma \\ \alpha \\ \beta \end{pmatrix}$, $\vec{n} = (F_x, F_y, F_z)$, $A = \begin{pmatrix} x, 1+x^2, 1 \\ y, 1+y^2, 1 \\ z, 1+z^2, 1 \end{pmatrix}$, $x=0, y=1, z=2$
- $Y_{i+1} = Y_i + k_i \Delta t (f(x_i, y_i))$, $2 \arctan x - x = 0$, $I = (1, 10)$
- $\iiint_M z dx dy dz = \int_0^{2\pi} \int_0^2 \left(\int_0^1 r^2 r dr \right) d\varphi dz$, $\cos \varphi = \frac{1}{\sqrt{2}}$, $\frac{1}{\sqrt{2}}$
- $x^2 - y + z = 1$, $x + y + z = 2$, $x_1 = \begin{pmatrix} 2p \\ -p \\ 0 \end{pmatrix}$, $16 - x^2 + 16y^2 - 4z > 0$
- $\chi \frac{\partial f}{\partial x} = 16 - x^2 + \cos^2 \beta + \cos^2 \gamma = 1$, $Y' = \frac{\sqrt{y}}{x+2} = 0$, $Y(0) = 1$
- $A+B+C=8$, $k_1 + k_2 \neq 0$, $p \neq 0$, $\lim_{x \rightarrow 0} \frac{e^{2x}-1}{5x} + \frac{2}{3}$, $\langle \beta, \gamma \rangle \in C$, $\text{grad} f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right)$
- $-3A - 7B + 2C = 10, 3$, $(1+e^x) y' = e^x \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 0$
- $-18A + 6B - 3C = 15$, $e^2 - xy z = e$, $A[0, e; 1]$, $Y(0) = 1$, $A = [1, 0, 3]$
- $\int_{-\pi/2}^{\pi/2} \sin^4 x \cdot \cos^2 x dx$, $y = \sqrt{x+1}$, $x = \log t$, $\int \frac{1}{3x^2 + 166x - 012} dx$, $\lim_{h \rightarrow \infty} \left(1 + \frac{2}{h}\right)^h$, $f(x) = 2^{-x} + 1$, $C = 0.005$



From rules to prediction

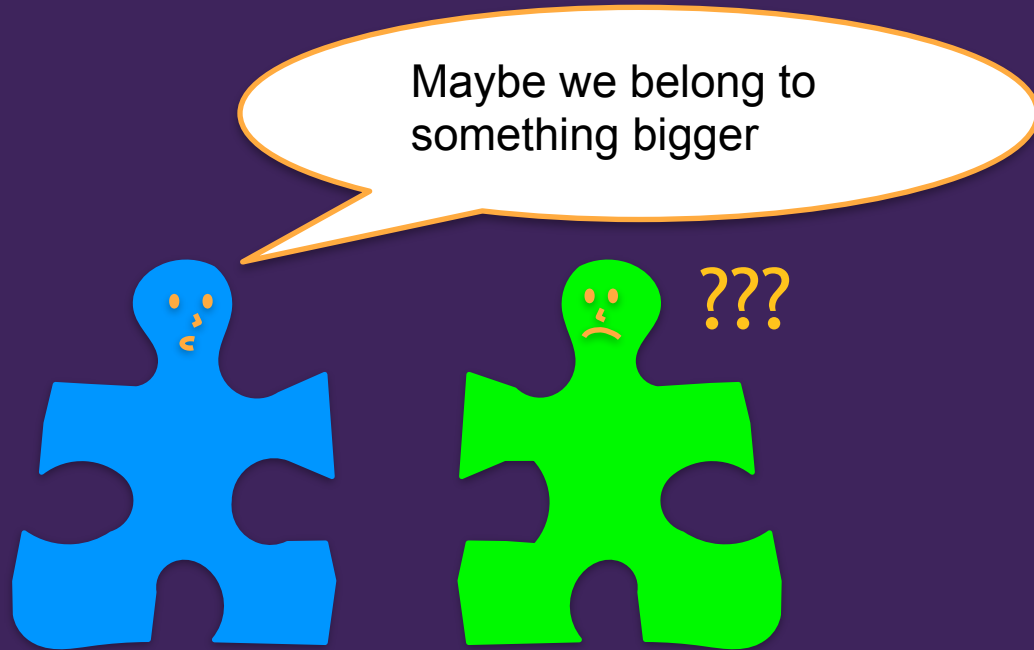
From data to rules

$\lim_{x \rightarrow 0} \frac{e^{2x}-1}{5x} + \frac{2}{3}$ $\lambda_2 = i\sqrt{14}$ $B = \begin{pmatrix} 2 & 1 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$ $\frac{2x}{x^2+2y^2} = 2 \frac{\sin x}{x} \leq \frac{x}{x} = 1$ $\delta(\rho) = \sqrt{10}\pi$
 $z = \frac{1}{x} \arcsin \frac{\sqrt{z}}{2}$ $\frac{\partial z}{\partial x} = 2, \frac{\partial z}{\partial y} = 0$ $\int_{\mathbb{R}} f(x, \sqrt{\frac{2x+1}{x+1}}) dx$ $x^4 + x^2 + y^3 + z^3 + xy^2 - t = 0$
 $x_1 = -1, x_2 = -1, x_3 = 7, p \in \mathbb{R}$
 $x_2 = \begin{pmatrix} x \\ y \\ z \end{pmatrix} C = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\lim_{n \rightarrow \infty} \frac{\sqrt{n^2+1+n}}{\sqrt{3n^2+2n-1}}$ $F_2 = 2xy^2 - 1 = 1$ $\eta_1 = \lambda^2 - 3\lambda + 1 + 0$
 $x \in \mathbb{R}$ $x_1 = \begin{pmatrix} \alpha + \beta + \gamma \\ \alpha \\ \beta \end{pmatrix}$ $\vec{n} = (F_x, F_y, F_z)$ $A = \begin{pmatrix} x & 1+x^2 & 1 \\ y & 1+y^2 & 1 \\ z & 1+z^2 & 1 \end{pmatrix}$ $x=0, y=1, z=2$
 $Y_{i+1} = Y_i + k_i \Delta t (f(t_i, Y_i))$
 $\iiint_M z dx dy dz = \int_0^{2\pi} \left(\int_0^1 \left(\int_0^1 r^2 dr \right) d\varphi \right) \cos \varphi = \frac{(1/3)(\pi/2)(1/4\pi)}{\sqrt{2} + \frac{1}{\sqrt{2}}}$
 $2 \arctan x - x = 0, I = (1, 10)$
 $x^2 - y + z = 1$ $x_1 = \begin{pmatrix} 2p \\ -p \\ 0 \end{pmatrix}$ $\frac{\partial f}{\partial x} = 16 - x^2 + 16y^2 - 4z > 0$ $1 \cdot Y' - \frac{Y}{x+2} = 0, Y(0) = 1$
 $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$
 $A+B+C=8$ $\frac{\partial f}{\partial x} = 16 - x^2 + 16y^2 - 4z > 0$ $\alpha, \beta, \gamma \in \mathbb{C}$ $g \circ df = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right)$
 $-3A - 7B + 2C = 10, 3$ $k_1 + k_2 \neq 0, \mu \neq 0$ $\lim_{x \rightarrow 0} \frac{e^{2x}-1}{5x} + \frac{2}{3}$ $(1+e^x) \eta = e^x \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 0$
 $-18A + 6B - 3C = 15$ $e^2 - xy^2 = e; A[0, e; 1]$ $Y(0) = 1$ $A = [1, 0, 3]$
 $\int_{-\pi/2}^{\pi/2} \sin^4 x \cdot \cos^2 x dx$ $y = \sqrt{x+1}, x = \log t$ $\int \frac{1}{3x^2 + 166x - 012} dx$ $\lim_{n \rightarrow \infty} \left(1 + \frac{2}{n}\right)^n$ $f(x) = 2^{-x} + 1, C = 0.005$



Best treatments

A big puzzle



The future

depends on maths